Simplicity in Complex Nuclei and the Quest for Symmetries:

First Test of the E(5/4) Bose-Fermi Critical Point Symmetry

R. F. Casten, M. S. Fetea, R. B. Cakirli, and D. D. Warner
I first met Dave in the early 1980’s when he and Rick were working on the spectrum of Er168, later published (1981) in a Phys Rev Letter with Davidson. This work caused a sensation in Copenhagen as it showed that one could describe strongly deformed nuclei within the framework of the Interacting Boson Model. Since that time, I had the pleasure of having many conversations with him, especially at the time he was working on M1 transitions in deformed nuclei, which he published in 1983, and then again when he was working with Rick on what later became known as the consistent –Q formalism. This paper has become a “classic” in nuclear physics.

Dave was one of those rare “complete” physicists, who can as easily do theory as experiment. He was very much attracted by simplicity and its manifestation “symmetry”. I cannot mention in a few words all the work that Dave did on the subject, ranging from boson symmetries to Bose-Fermi and super-symmetries. In some of these papers he was the sole author. In others, he collaborated with Rick, and in others yet with Piet. Symmetry was again the dominant theme of these collaborations. I have here in mind the work on Pt195 related to super-symmetry (with Rick) and the work on the spin-isospin interacting boson model (with Piet).

The entire field of Nuclear Physics owes Dave a lot. I particularly want to express gratitude to his memory for all he did for the field.

The last time I saw Dave was last spring in New Haven, when he came for a visit to the Nuclear Structure Lab. We sat at dinner together with his wife, Lynne. He was planning many experiments and testing new ideas. He said, “What better than looking again at super-symmetry?” He was referring to the paper that he and Rick were planning to write on the latest development in symmetries in nuclei, called E(5/4).

To his wife Lynne, his family and all his friends I express my sincere condolences. We will all miss Dave.  

Franco Iachello
Dear Rick,

I know that you will be attending a Nuclear Physics Symposium in honor of David Warner. I was very shocked and deeply saddened to hear of his sudden and premature death. His physics was deep and influential, and he was a wonderful person to interact with both in science and also socially. I shall miss him greatly.

Witek
Themes and challenges of Modern Science

• Complexity out of simplicity

   How the world, with all its apparent complexity and diversity, can be constructed out of a few elementary building blocks and their interactions

• Simplicity out of complexity

   How the world of complex systems can display such remarkable regularity and simplicity

• Understanding the nature of the physical universe

• Manipulating nature for the benefit of mankind

Nuclei: Two-fluid, many-body, strongly-interacting, quantal systems provide wonderful laboratories for frontier research in all four areas
Simplicity out of complexity.

Astonishing simplicity in a complex many-body object.

The graph shows a spectrum with energy (keV) on the x-axis and counts on the y-axis. The peaks correspond to different values of $J$: $J$, $J + 2$, $J + 4$, $J + 6$, and $J + 8$.
Correlations of Collective Observables

\[ R_{4/2} \equiv \frac{E(4^+_1)}{E(2^+_1)} \]

\[ Z = 38-82 \]

\[ R_{4/2} > 2.05 \]
Approaches to Nuclear Structure

**Microscopic** – Approximate solutions to real nuclei

- Effective Interactions
- Ab initio, No core, Monte Carlo
- Density Functional Theory

Enormously complex, numerically intensive. However, revolutionary advances, greatly enhanced ability to predict wide variety of nuclei → promise of a comprehensive theory

**Macroscopic** – Exact solutions to ideal nuclei

Geometric symmetries. Simple patterns, quantum nos., Selection rules

- Analytic, Intuitive understanding -- WHAT symmetries?
- Challenge to microscopy – Why THESE symmetries, which nuclei, why in THESE nuclei?
Most nuclei do not exhibit the idealized symmetries but rather lie in transitional regions. Trajectories of structural evolution
Phase Transitions in Nuclei

• Critical Point Symmetries: $X(5)$, $E(5)$
Nuclear Shape Evolution

Vibrational Region

Transitional Region

Rotational Region

\[ V(\beta) \]

\[ \beta \]

\[ E_n = n\hbar\omega \]

Critical Point

New analytical solutions, E(5) and X(5)

\[ E_J \sim J(J + 1) \]
Dynamic Symmetries at the Critical Point

- approximate potential at phase transition with infinite square well
- solve Bohr Hamiltonian for the new potential
- analytic solution in terms of zeros of special Bessel functions
- predictions for energies and electromagnetic transition probabilities

Critical Point Symmetries
First Order Phase Transition – Phase Coexistence

Energy surface changes with valence nucleon number

Bessel equation

\[ \ddot{\xi} + \frac{\xi'}{z} + \left[ 1 - \frac{v^2}{z^2} \right] \xi = 0; \quad \ddot{\xi} (\beta_w) = 0. \]

\[ v = \left( \frac{L(L+1)}{3} + \frac{9}{4} \right)^{1/2} \]

Iachello
New Paradigm for Nuclear Structure
Critical Point Phase Transitional Nuclei

E(0\textsubscript{2})/E(2)=5.67

E(4)/E(2)=2.91

Zamfir

X(5)

\(^{152}\text{Sm}\)
FIG. 1. Schematic representation of the lowest portion of the spectrum of the five-dimensional infinite well [E(5) symmetry]. Energies are in units of the energy of the first excited state, $E_{2,1}$. $B(E2)$ values are in units of $B(E2; 2_{1,1} \rightarrow 0_{1,0}) = 100$. 
E(5) and $^{134}\text{Ba}$

Five dimensional square well $E(5)$ coupled to a $j=3/2$ particle (neutron or proton)

Describes odd nuclei at the critical value of spherical to $\gamma$-unstable transition


$^{135}\text{Ba}$

$^{134}\text{Ba}$

$+ n d_{3/2}$
U(5) even-even  O(6)  "U(5)"  odd-A  "O(6)"

Alonso et al, PR C
$^{135}\text{Ba}$
\[ k = -0.5 \]

\[
\begin{align*}
E \text{ (MeV)} & \quad \xi = 2^+ \\
\left[0, \frac{1}{2}\right] & \quad \xi = 2^+ \\
\left[2, \frac{5}{2}\right] & \quad \xi = 1^- \\
\left[1, \frac{1}{2}\right] & \quad \xi = 1^- \\
\left[0, \frac{1}{2}\right] & \quad \xi = 1^+ \\
E (5/4) & \quad \xi = 1^+ \\
\end{align*}
\]

\[
\frac{B(E2; \left(\frac{3}{2}^+\right)_3 \rightarrow \left(\frac{5}{2}^+\right)_1)}{B(E2; \left(\frac{3}{2}^+\right)_3 \rightarrow \left(\frac{1}{2}^+\right)_1)} \leq 0.44
\]
$^{135}\text{Ba}$

Experiment  
Shell Model
Principal Collaborators

Mirela Fetea

R. Burcu Cakirli

Dave Warner

Dave, Thanks for everything !!!
Where does it work?

X(5)

E(5)
Structural Evolution, Phase Transitions, and Critical Point Nuclei
No shell closure for $N=8$ and 20 for drip-line nuclei; new shells at 14, 16, 32...
Searching in new regions

Studies so far have concentrated on stable or near stable nuclei

Still a variety of regions which could provide a testing ground or possibly new structures
<table>
<thead>
<tr>
<th></th>
<th>58</th>
<th>56</th>
<th>54</th>
<th>52</th>
<th>50</th>
<th>48</th>
<th>46</th>
<th>44</th>
<th>42</th>
<th>40</th>
<th>38</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>52</td>
</tr>
<tr>
<td>58</td>
<td>3.06</td>
<td>2.93</td>
<td>2.80</td>
<td>2.69</td>
<td>2.56</td>
<td>2.42</td>
<td>1.58</td>
<td>1.68</td>
<td>1.75</td>
<td>1.79</td>
<td>1.84</td>
<td>1.89</td>
</tr>
<tr>
<td>56</td>
<td>2.96</td>
<td>2.89</td>
<td>2.83</td>
<td>2.78</td>
<td>2.69</td>
<td>2.52</td>
<td>2.35</td>
<td>2.24</td>
<td>2.16</td>
<td>2.07</td>
<td>2.07</td>
<td>2.05</td>
</tr>
<tr>
<td>54</td>
<td>2.33</td>
<td>2.40</td>
<td>2.47</td>
<td>2.50</td>
<td>2.48</td>
<td>2.42</td>
<td>2.39</td>
<td>2.38</td>
<td>2.39</td>
<td>2.38</td>
<td>2.38</td>
<td>2.33</td>
</tr>
<tr>
<td>52</td>
<td>2.09</td>
<td>2.00</td>
<td>1.99</td>
<td>2.07</td>
<td>2.09</td>
<td>2.07</td>
<td>2.04</td>
<td>2.01</td>
<td>1.94</td>
<td>1.72</td>
<td>1.54</td>
<td>1.67</td>
</tr>
<tr>
<td>50</td>
<td>1.54</td>
<td>1.67</td>
<td>1.75</td>
<td>1.81</td>
<td>1.79</td>
<td>1.68</td>
<td>1.84</td>
<td>1.85</td>
<td>1.87</td>
<td>1.88</td>
<td>1.86</td>
<td>1.72</td>
</tr>
<tr>
<td>48</td>
<td>1.79</td>
<td>2.11</td>
<td>2.27</td>
<td>2.36</td>
<td>2.38</td>
<td>2.33</td>
<td>2.29</td>
<td>2.30</td>
<td>2.38</td>
<td>2.39</td>
<td>2.38</td>
<td>2.33</td>
</tr>
<tr>
<td>46</td>
<td>1.79</td>
<td>2.12</td>
<td>2.23</td>
<td>2.32</td>
<td>2.33</td>
<td>2.27</td>
<td>2.32</td>
<td>2.48</td>
<td>2.65</td>
<td>2.75</td>
<td>2.76</td>
<td>2.73</td>
</tr>
<tr>
<td>44</td>
<td>1.82</td>
<td>2.14</td>
<td>2.27</td>
<td>2.32</td>
<td>2.33</td>
<td>2.26</td>
<td>2.50</td>
<td>2.92</td>
<td>3.05</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
</tr>
<tr>
<td>42</td>
<td>1.81</td>
<td>2.09</td>
<td>1.92</td>
<td>2.12</td>
<td>2.51</td>
<td>2.92</td>
<td>3.05</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
</tr>
<tr>
<td>40</td>
<td>1.60</td>
<td>1.60</td>
<td>1.63</td>
<td>1.51</td>
<td>2.65</td>
<td>3.15</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
</tr>
<tr>
<td>38</td>
<td>1.99</td>
<td>2.05</td>
<td>3.01</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>58</th>
<th>56</th>
<th>54</th>
<th>52</th>
<th>50</th>
<th>48</th>
<th>46</th>
<th>44</th>
<th>42</th>
<th>40</th>
<th>38</th>
<th>Z/N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>18</td>
<td>2.32</td>
<td>2.28</td>
<td>2.24</td>
<td>2.16</td>
<td>2.04</td>
<td>1.94</td>
<td>1.72</td>
<td>1.63</td>
<td>1.72</td>
<td>1.72</td>
<td>1.72</td>
</tr>
</tbody>
</table>

P~2.5
The Symmetry Triangle of Nuclear Structure
Dynamical Symmetries

\[ \begin{align*}
&\text{U(5)} \\
&\text{Sph.} \\
&\text{Vibrator} \\
&\text{O(6)} \\
&\text{Sph.} \\
&\text{Deformed} \\
&\text{SU(3)} \\
&\text{Prolate Rotor} \\
\end{align*} \]
Original E(5) fitted to 134Ba
The Symmetry Triangle -- a structural mapping

Dynamical Symmetries

Phase/shape Transitions

Most nuclei do not exhibit the idealized symmetries but rather lie in transitional regions. Trajectories of structural evolution.